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Fuzzy Total Coloring and Chromatic Number of Strong Simple Fuzzy Graph

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ABSTRACT

In this paper, we discuss about the total coloring families of Star fuzzy graph, Wheel fuzzy graph, Line fuzzy graph. We obtain the total coloring chromatic number, order and Size of the strong simple fuzzy graph.

Keywords: Fuzzy Graph, Strong Fuzzy graph, Wheel fuzzy graph, Line fuzzy graph Total Coloring, Chromatic Number, Order, Size, etc……

I. INTRODUCTION

Fuzzy Graph was first introduced by Azriel Rosenfeld [1] in 1975 .He introduced and examined such concept as paths, Connectedness, Clusters, bridges, Cut vertices, forests and trees. Computing chromatic sum of an arbitrary Graph was discussed by Kubica (1989), known as NP – Complete problem. Eslahchi and onagh defined Fuzzy coloring of Fuzzy Graph in 2004, and Fuzzy Total coloring and Chromatic Number of a Complete Fuzzy Graph [2] in 2013. Graph Coloring has applications to many real world problems like scheduling, telecommunications, bioinformatics etc. In 1965 Behzad and Vizing have found independently a new concept of graph coloring called total coloring. Fuzzy Total coloring of Fuzzy Graphs was discussed by Lavanya .S and Sattanathan.R. [3] in 2009. An edge coloring of a simple Graph G is an assignment of colors to the edges so that no two incident edges receive the same color. The edge chromatic number $x'(G)$ was defined by Clolin Mcdiarmid and Bruce Reed [4] in 1993. Star – in - coloring of some new class of Graphs was discussed by Sudha.S and Kanniga.V. [5], Order and Size in fuzzy Graph was discussed by Nagoor Gani A and basher Ahamed.M [6] in 2003 ..In this paper we discuss about fuzzy total coloring and chromatic number of a simple fuzzy graph 1.

II. PRELIMINARIES

Definition 2.1:

Fuzzy Graph with S as the underlying set is a pair $G=(\sigma,\mu)$ where $\sigma:V\rightarrow[0,1]$ is a fuzzy subset and $\mu: V \times V \rightarrow [0,1]$ is a fuzzy relation on the subset σ such that $\mu(x,y) \leq \min\{\sigma(x), \sigma(y)\}$.

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Definition 2.2:

Let (σ, μ) be fuzzy sub graph of G = (V, X) Then (σ, μ) is called a strong fuzzy graph H of G if μ (u, v) = σ (u) ∧ σ (v) for all (u, v) ϵ X.

Definition 2.3:

Let G: (σ, μ) be a fuzzy graph, the order of G is defined as O(G) = $\sum_{u\in V}\sigma(u)$.

Definition 2.4:

Let G: (σ, μ) be a fuzzy graph the size of G is defined as S(G) = $\sum_{u,v\in V}\mu(u,v)$.

Definition 2.5:

A fuzzy star graph $s_{1,n}$ consists of two no edge sets V and U with ∣ V ∣ =1 and ∣ U ∣ > 1 such that μ (v , u_i) > 0 and (u_i, u_{i+1}) = 0 , 1 ≤ I ≤ n.

Definition 2.6:

A wheel graph with fuzzy labeling is called a fuzzy wheel graph.

Definition 2.7:

A family Γ = $\{\gamma_1,\gamma_{2,}\ldots\ldots\gamma_n\}$ of fuzzy sets on V is called a k-fuzzy coloring of Ĝ = (V, σ, μ) if

a) vΓ = σ

b) $\gamma_i \wedge \gamma_j = 0$

c) For every strong edge uv of G, min $\{\gamma_i(u), \gamma_j(v)\}$ = 0 (1 ≤ i ≤ k)

Definition 2.8:

A family Γ = $\{\gamma_1,\gamma_{2,}\ldots\ldots\gamma_k\}$ of fuzzy sets on V∪E is called a k-fuzzy total coloring of Ĝ = (V, σ, μ) if

(a) $\text{Max}_i\{\gamma_i(v)\}$ = (v) for all v∈V and $\max_i\{\gamma_i(u), \gamma_j(v)\} = \mu(uv)$ for all edges uv_s

(b) $\gamma_i \wedge \gamma_j = 0$

For every adjacent vertices u, v of min $\{\gamma_i(u), \gamma_j(v)\} = 0$ and for every incident edges min$\{\gamma_i(v_j,v_k)/ v_jv_k$ are set of incident edges from the vertex $v_j\} = 0$,j = 1,2,…. $|v|$.

III. IN THIS SECTION WE DISCUSS THE ORDER, SIZE AND TOTAL COLORING OF SIMPLE STRONG STAR FUZZY GRAPH

Theorem: 3.1

The fuzzy total coloring of strong star fuzzy graph satisfies the following condition S(G)≤S(G').

Proof:

Let G: (v, σ, μ) be a star fuzzy graph,

The Size of star fuzzy graph is then given by

$$S(G)=\sum_{(u,v_i)\in E}\mu(u,v_i)$$

Where

$$\mu(u,v_i)\le \sigma(u)\sigma(v_i) \text{ ------------------ (1)}$$

Let G':(v'σ'μ') be a star strong fuzzy graph. And the size of a strong star fuzzy graph is,

$$S(G')=\sum(\mu'(u,v_i)$$

Where

$$\mu'(u,v_i)=\sigma(u)\sigma(v_i) \text{ ----------------(2)}$$

From (1) &(2)

$$\mu(u,v_i)\le\mu'(u,v_i)$$

$$\sum\mu(u,v_i)\le\sum\mu'(u,v_i)$$

S(G)≤S(G').

Example:

We can illustrate the theorem with the following example:

Consider the star fuzzy graph G with vertices $V_1 = 0.0$, $V_2 = 0.6$............$V_9 = 0.3$

Example 1:

Consider simple fuzzy graph $S_{(1.8)}$ with n = 9 vertices

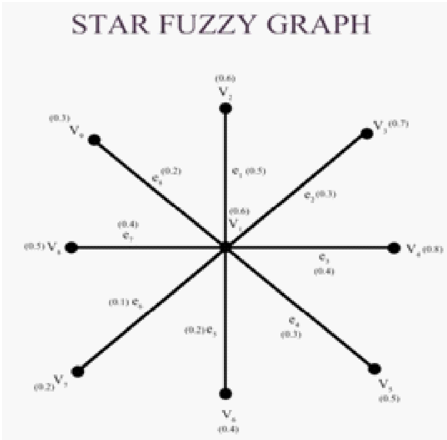

Fig 3.1 Star Fuzzy Graph G

O(G) = 4, S(G) = 2.4

Then the strong star fuzzy

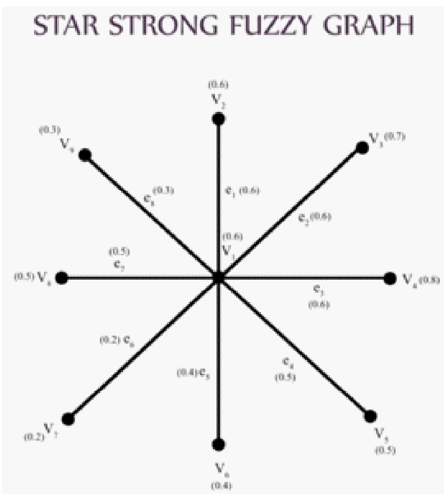

Fig 3.2 Strong Stat Fuzzy Graph G′

O(G)= 4, S(G') = 3.7

Remark: 3.2

The order of a Star fuzzy graph is equal to the order of its strong star fuzzy graph.

Theorem: 3.3

The fuzzy total coloring of strong star fuzzy graph n-1 where n is the number of vertices.

i.e.) $\chi_s''(G') = n-1$

Proof:

Let H be a strong star fuzzy graph with n vertices. They have n -1 edges, For any edge, there are two end nodes.

The edge incident to those nodes and the edge cannot be included in the same color such a collection form n-1 colors for vertices.

$$\chi_s''(G') = n-1$$

Example 1:

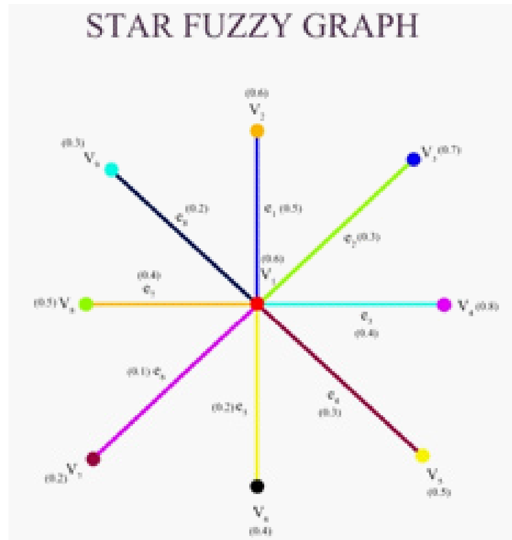

Fig 3.3 Star Fuzzy Graph

Example 2:

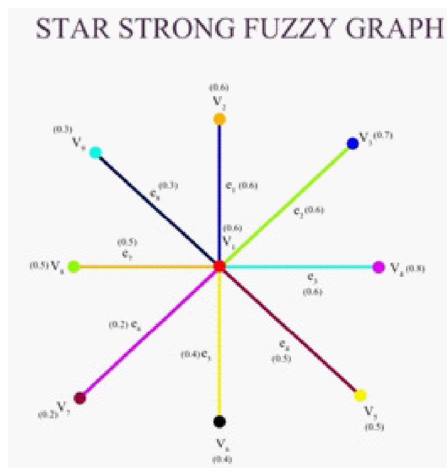

Fig 3.4 Star Strong Fuzzy Graph

IV. IN THIS SECTION WE DISCUSS TOTAL COLORING OF SIMPLE STRONG LINE FUZZY GRAPH

Theorem: 4.1

For fuzzy line graph G;(V, σ, μ) $\chi(G) = \chi^f(G)$.

Proof:

Let G = (V, σ, μ) be a fuzzy line graph of n vertices $\{v_1, v_2, \ldots .. v_n\}$ and n-1 edges $\{e_1, e_2 \ldots .. e_{n-1}\}$.

Let $\chi^f(G) = k$

$\Leftrightarrow \Gamma = \{\gamma_1, \gamma_2, \ldots\ldots \gamma_k\}$ is k – fuzzy coloring and let C_j be the color assigned to vertices in, γ_j^*, j = 1, 2, …k.

$\Leftrightarrow \{\gamma_1, \gamma_2, \ldots\ldots \gamma_k\}$ is a family of fuzzy sets

Where

$$\gamma_j(v_i) = \{v_j, \sigma(v_i)\} \cup \{v_i, \sigma(v_i) / \mu(v_i, v_j)\} = 0 \;\; i \neq j$$

which follows from (i) and (ii) of Definition 2.8

Also $\bigcup_{i=1}^{k} \gamma_j^* = V$ and $\gamma_i^* \cap \gamma_j^* = \phi,\ i \neq j$, which follows from (ii) of Definition 2.8

$\Leftrightarrow \gamma_j^*$ is an independent set of vertices.

(i.e. no two vertices in γ_j^* are adjacent) for each j = 1, 2, … k.

$\Leftrightarrow \chi(G^*) = \chi(G_t) = k$

Where $t = \min\{\alpha / \alpha \in L\} = \max\{X_\alpha / \alpha \in L\}$

$a_i <\propto_j, \chi_{\propto_i} \geq, \chi_{\propto_j}$

$\chi(G) = \chi^f(G)$.

Example: 4.1

Fuzzy Line Graph

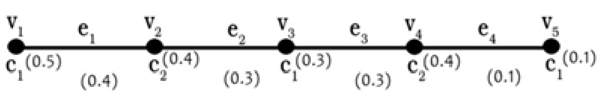

Fig 4.1 L_9 Fuzzy Line Graph

V. IN THIS SECTION WE DISCUSS TOTAL COLORING OF SIMPLE STRONG WHEEL FUZZY GRAPH

Remark: 5.1

Every fuzzy wheel graph is not a strong fuzzy wheel graph .

Proof:

"A wheel graph with fuzzy labeling is called a fuzzy wheel graph".

"A Graph G = (σ, μ) is said to be a fuzzy labeling if σ: V → [0, 1] and μ:v x v → [0, 1] is bijective so that the member ship value of edges and vertices are distinct and μ (u, v) = σ (u) ∧ σ (v) for all (u, v) ϵ V."

While considering a normal wheel fuzzy graph and strong wheel fuzzy graph that the labeling value of normal wheel fuzzy graph varies from the fuzzy labeling of the strong wheel fuzzy graph.

So that, I conclude that wheel fuzzy does not exists to the strong wheel fuzzy graph.

Example: 5.1

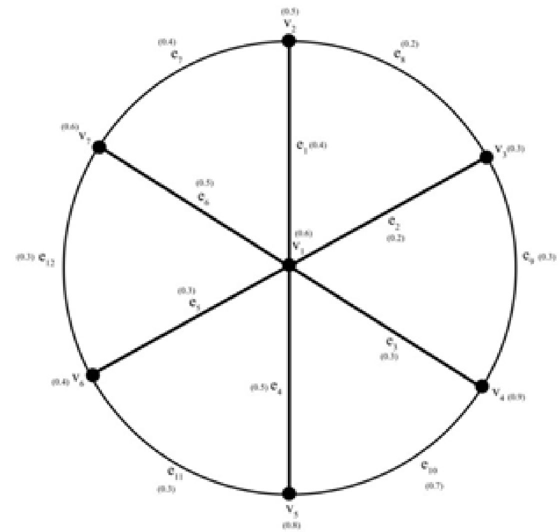

Fig 5.1 Wheel Fuzzy Graph

Example: 5.2

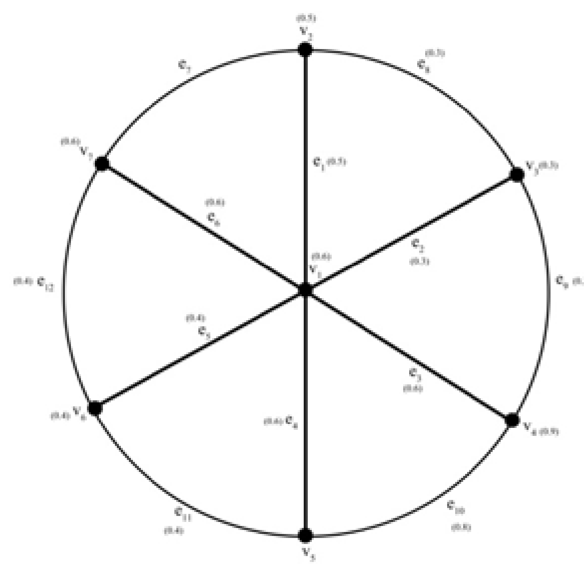

Fig 5.2 Strong Wheel Fuzzy Graph

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